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Confronting Finite Unified Theories with Low-Energy Phenomenology

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Abstract

Finite Unified Theories (FUTs) are $N = 1$ supersymmetric Grand Unified Theories that can be made all-loop finite. The requirement of all-loop finiteness leads to a severe reduction of the free parameters of the theory and, in turn, to a large number of predictions. FUTs are investigated in the context of low-energy phenomenology observables. We present a detailed scanning of the all-loop finite $SU(5)$ FUTs, where we include the theoretical uncertainties at the unification scale and we apply several phenomenological constraints. Taking into account the restrictions from the top and bottom quark masses, we can discriminate between different models. Including further low-energy constraints such as B physics observables, the bound on the lightest Higgs boson mass and the cold dark matter density, we determine the predictions of the allowed parameter space for the Higgs boson sector and the supersymmetric particle spectrum of the selected model.

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1 Introduction

A large and sustained effort has been done in the recent years aiming to achieve a unified description of all interactions. Out of this endeavor two main directions have emerged as the most promising to attack the problem, namely, the superstring theories and non-commutative geometry. The two approaches, although at a different stage of development, have common unification targets and share similar hopes for exhibiting improved renormalization properties in the ultraviolet(UV) as compared to ordinary field theories. Moreover the two frameworks came closer by the observation that a natural realization of non-commutativity of space appears in the string theory context of D-branes in the presence of a constant background antisymmetric field [1]. However, despite the importance of having frameworks to discuss quantum gravity in a self-consistent way and possibly to construct there finite theories, it is very interesting to search for the minimal realistic framework in which finiteness can take place. In addition the main goal expected from a unified description of interactions by the particle physics community is to understand the present day large number of free parameters of the Standard Model (SM) in terms of a few fundamental ones. In other words, to achieve *reduction of couplings* at a more fundamental level. A complementary, and certainly not contradicting, program has been developed [2–4] in searching for a more fundamental theory possibly at the Planck scale called Finite Unified Theories (FUTs), whose basic ingredients are field theoretical Grand Unified Theories (GUTs) and supersymmetry (SUSY), but its consequences certainly go beyond the known ones.

Finite Unified Theories are $N = 1$ supersymmetric GUTs which can be made finite even to all-loop orders, including the soft supersymmetry breaking sector. The method to construct GUTs with reduced independent parameters [5, 6] consists of searching for renormalization group invariant (RGI) relations holding below the Planck scale, which in turn are preserved down to the GUT scale. Of particular interest is the possibility to find RGI relations among couplings that guarantee finiteness to all-orders in perturbation theory [7, 8]. In order to achieve the latter it is enough to study the uniqueness of the solutions to the one-loop finiteness conditions [7–9]. The constructed *finite unified* $N = 1$ supersymmetric GUTs, using the above tools, predicted correctly from the dimensionless sector (Gauge-Yukawa unification), among others, the top quark mass [2]. The search for RGI relations and finiteness has been extended to the soft supersymmetry breaking sector (SSB) of these theories [10–19], which involves parameters of dimension one and two. Eventually, the full theories can be made all-loop finite and their predictive power is extended to the Higgs sector and the SUSY spectrum. This, in turn, allows to make predictions for low-energy precision and astrophysical observables. The purpose of the present article is to do an exhaustive search of these latter predictions of the SU(5) finite models, taking into account the restrictions resulting from the low-energy observables. Then we present the predictions of the models under study for the parameter space that is still allowed after taking the phenomenological restrictions into account. Here we focus on the Higgs boson sector and the SUSY spectrum.

In our search we consider the restrictions imposed on the parameter space of the models due to the following observables: the 3rd generation quark masses, rare b decays, $\mathrm{BR}(b \to s\gamma)$ and $\mathrm{BR}(B_s \to \mu^+\mu^-)$, as well as the mass of the lightest $\mathcal{CP}$-even Higgs boson, M_h. Present data on these observables already provide interesting information about the allowed SUSY mass scales. The non-discovery of the Higgs boson at LEP [20, 21] excludes a part of the

otherwise allowed parameter space. However the non-discovery of supersymmetric particles at LEP does not impose any restrictions on the parameter space of the models, given that their SUSY spectra turn out to be very heavy anyway. An important further constraint is provided by the density of dark matter in the Universe, which is tightly constrained by WMAP and other astrophysical and cosmological data [22], assuming that the dark matter consists largely of neutralinos [23]. We also briefly discuss the implication from the anomalous magnetic moment of the muon, $(g-2)_\mu$. Other recent analyses of GUT based models confronted with low-energy observables and dark matter constraints can be found in Refs. [24, 25].

In this context we first review the sensitivity of each observable to indirect effects of supersymmetry, taking into account the present experimental and theoretical uncertainties. Later on we investigate the part of parameter space in the FUT models under consideration that is still allowed taking into account all low-energy observables.

In Sect. 2 of the paper we review the conditions of finiteness in $N = 1$ SUSY gauge theories. The consequences of finiteness for the soft SUSY-breaking terms are discussed in Sect. 3. The two $SU(5)$ FUT models that emerge are briefly presented in Sect. 4. In Sect. 5 we discuss different precision observables, including the cold dark matter constraint. Sect. 6 contains the analysis of the parts of parameter space that survive all constraints and the final predictions of the models. We conclude with Sect. 7.

2 Reduction of Couplings and Finiteness in $N = 1$ SUSY Gauge Theories

Here we review the main points and ideas concerning the *reduction of couplings* and *finiteness* in $N = 1$ supersymmetric theories. A RGI relation among couplings g_i, $\Phi(g_1, \cdots, g_N)$=0, has to satisfy the partial differential equation $\mu \; d\Phi/d\mu \; = \; \sum_{i=1}^{N} \beta_i \, \partial\Phi/\partial g_i$=0, where β_i is the β-function of g_i. There exist $(N-1)$ independent Φ's, and finding the complete set of these solutions is equivalent to solve the so-called reduction equations (REs) [5], $\beta_g \, (dg_i/dg) = \beta_i \;, \; i = 1, \ldots, N$, where g and β_g are the primary coupling (in favor of which all other couplings are reduced) and its β-function. Using all the $(N-1) \, \Phi$'s to impose RGI relations, one can in principle express all the couplings in terms of a single coupling g. The complete reduction, which formally preserves perturbative renormalizability, can be achieved by demanding a power series solution, whose uniqueness can be investigated at the one-loop level.

Finiteness can be understood by considering a chiral, anomaly free, $N = 1$ globally supersymmetric gauge theory based on a group G with gauge coupling constant g. The superpotential of the theory is given by

$$W = \frac{1}{2} \, m^{ij} \, \Phi_i \, \Phi_j + \frac{1}{6} \, C^{ijk} \, \Phi_i \, \Phi_j \, \Phi_k \;, \tag{1}$$

where m^{ij} (the mass terms) and C^{ijk} (the Yukawa couplings) are gauge invariant tensors and the matter field Φ_i transforms according to the irreducible representation R_i of the gauge group G.

The one-loop β-function of the gauge coupling g is given by

$$\beta_g^{(1)} \;=\; \frac{dg}{dt} = \frac{g^3}{16\pi^2}\,[\,\sum_i \ell(R_i) - 3\,C_2(G)\,]\;, \tag{2}$$

where $\ell(R_i)$ is the Dynkin index of R_i and $C_2(G)$ is the quadratic Casimir of the adjoint representation of the gauge group G. The β-functions of C^{ijk}, by virtue of the non-renormalization theorem, are related to the anomalous dimension matrix γ_i^j of the matter fields Φ_i as

$$\beta_C^{ijk} = \frac{d}{dt}\,C^{ijk} = C^{ijp}\,\sum_{n=1}\frac{1}{(16\pi^2)^n}\,\gamma_p^{k(n)} + (k \leftrightarrow i) + (k \leftrightarrow j)\;. \tag{3}$$

At one-loop level γ_i^j is given by

$$\gamma_i^{j(1)} = \frac{1}{2}C_{ipq}\,C^{jpq} - 2\,g^2\,C_2(R_i)\delta_i^j\;, \tag{4}$$

where $C_2(R_i)$ is the quadratic Casimir of the representation R_i, and $C^{ijk} = C^*_{ijk}$.

All the one-loop β-functions of the theory vanish if the β-function of the gauge coupling $\beta_g^{(1)}$, and the anomalous dimensions $\gamma_i^{j(1)}$, vanish, i.e.

$$\sum_i \ell(R_i) = 3C_2(G)\;,\;\; \frac{1}{2}C_{ipq}C^{jpq} = 2\delta_i^j g^2 C_2(R_i). \tag{5}$$

A very interesting result is that the conditions (5) are necessary and sufficient for finiteness at the two-loop level [9, 13].

The one- and two-loop finiteness conditions (5) restrict considerably the possible choices of the irreducible representations R_i for a given group G as well as the Yukawa couplings in the superpotential (1). Note in particular that the finiteness conditions cannot be applied to the supersymmetric standard model (SSM). The presence of a $U(1)$ gauge group, whose $C_2[U(1)] = 0$, makes impossible to satisfy the condition (5). This leads to the expectation that finiteness should be attained at the grand unified level only, the SSM being just the corresponding low-energy, effective theory.

The finiteness conditions impose relations between gauge and Yukawa couplings. Therefore, we have to guarantee that such relations leading to a reduction of the couplings hold at any renormalization point. The necessary, but also sufficient, condition for this to happen is to require that such relations are solutions to the reduction equations (REs) to all orders. The all-loop order finiteness theorem of [7] is based on: (a) the structure of the supercurrent in $N = 1$ SYM and on (b) the non-renormalization properties of $N = 1$ chiral anomalies. Alternatively, similar results can be obtained [8, 26] using an analysis of the all-loop NSVZ gauge beta-function [27].

3 Soft supersymmetry breaking and finiteness

The above described method of reducing the dimensionless couplings has been extended [10] to the soft supersymmetry breaking (SSB) dimensionful parameters of $N = 1$ supersymmetric

theories. In addition it was found [11] that RGI SSB scalar masses in general Gauge-Yukawa unified models satisfy a universal sum rule at one-loop, which was subsequently extended first up to two-loops [3] and then to all-loops [12].

To be more specific, consider the superpotential given by (1) along with the Lagrangian for SSB terms

$$-\mathcal{L}_{\rm SB} = \frac{1}{6}\,h^{ijk}\,\phi_i\phi_j\phi_k + \frac{1}{2}\,b^{ij}\,\phi_i\phi_j + \frac{1}{2}\,(m^2)^j_i\,\phi^{*\,i}\phi_j + \frac{1}{2}\,M\,\lambda\lambda + \text{h.c.}\,, \tag{6}$$

where the ϕ_i are the scalar parts of the chiral superfields Φ_i , λ are the gauginos and M their unified mass. Since we would like to consider only finite theories here, we assume that the one-loop β-function of the gauge coupling g vanishes. We also assume that the reduction equations admit power series solutions of the form $C^{ijk} = g\,\sum_{n=0}\,\rho^{ijk}_{(n)} g^{2n}$. According to the finiteness theorem of ref. [7], the theory is then finite to all orders in perturbation theory, if, among others, the one-loop anomalous dimensions $\gamma_i^{j(1)}$ vanish. The one- and two-loop finiteness for h^{ijk} can be achieved [9, 13] by imposing the condition

$$h^{ijk} = -MC^{ijk} + \cdots = -M\rho^{ijk}_{(0)}\,g + O(g^5)\,. \tag{7}$$

In addition, it was found [3] that one and two-loop finiteness requires that the following two-loop sum rule for the soft scalar masses has to be satisfied

$$\frac{(\ m_i^2 + m_j^2 + m_k^2\ )}{MM^\dagger} = 1 + \frac{g^2}{16\pi^2}\,\Delta^{(2)} + O(g^4)\,, \tag{8}$$

where $\Delta^{(2)}$ is the two-loop correction,

$$\Delta^{(2)} = -2\sum_i \left[\left(\frac{m_i^2}{MM^\dagger}\right) - \left(\frac{1}{3}\right)\right]\ \ell(R_i)\,, \tag{9}$$

which vanishes for the universal choice [13], as well as in the models we consider in the next section. Furthermore, it was found [14] that the relation

$$h^{ijk} = -Mg(C^{ijk})' \equiv -Mg\frac{dC^{ijk}(g)}{d\ln g}\,, \tag{10}$$

among couplings is all-loop RGI. Moreover, the progress made using the spurion technique leads to all-loop relations among SSB β-functions [4, 14] and [16–19], which allowed to find the all-loop RGI sum rule [12] in the Novikov-Shifman-Vainstein-Zakharov scheme [27].

4 Finite Unified Theories

Finite Unified Theories (FUTs) have always attracted interest for their intriguing mathematical properties and their predictive power. One very important result is that the one-loop finiteness conditions (5) are sufficient to guarantee two-loop finiteness [28]. A classification of possible one-loop finite models was done independently by several authors [29]. The first one and two-loop finite $SU(5)$ model was presented in [30], and shortly afterwards the conditions for finiteness in the soft SUSY-breaking sector at one-loop [9] were given. In [31] a one

and two-loop finite $SU(5)$ model was presented where the rotation of the Higgs sector was proposed as a way of making it realistic. The first all-loop finite theory was studied in [2], without taking into account the soft breaking terms. Finite soft breaking terms and the proof that one-loop finiteness in the soft terms also implies two-loop finiteness was done in [13]. The inclusion of soft breaking terms in a realistic model was done in [33] and their finiteness to all-loops studied in [34], although the universality of the soft breaking terms lead to a charged LSP. This fact was also noticed in [35], where the inclusion of an extra parameter in the Higgs sector was introduced to alleviate it. The derivation of the sum-rule in the soft supersymmetry breaking sector and the proof that it can be made all-loop finite were done in [12, 30, 31, 36], allowing thus for the construction of all-loop finite realistic models.

From the classification of theories with vanishing one-loop gauge β function [29], one can easily see that there exist only two candidate possibilities to construct $SU(5)$ GUTs with three generations. These possibilities require that the theory should contain as matter fields the chiral supermultiplets $\mathbf{5}$, $\overline{\mathbf{5}}$, $\mathbf{10}$, $\overline{\mathbf{5}}$, $\mathbf{24}$ with the multiplicities $(6, 9, 4, 1, 0)$ and $(4, 7, 3, 0, 1)$, respectively. Only the second one contains a $\mathbf{24}$-plet which can be used to provide the spontaneous symmetry breaking (SB) of $SU(5)$ down to $SU(3)\times SU(2)\times U(1)$. For the first model one has to incorporate another way, such as the Wilson flux breaking mechanism to achieve the desired SB of $SU(5)$ [2]. Therefore, for a self-consistent field theory discussion we would like to concentrate only on the second possibility.

The particle content of the models we will study consists of the following supermultiplets: three $(\overline{\mathbf{5}}+\mathbf{10})$, needed for each of the three generations of quarks and leptons, four $(\overline{\mathbf{5}}+\mathbf{5})$ and one $\mathbf{24}$ considered as Higgs supermultiplets. When the gauge group of the finite GUT is broken the theory is no longer finite, and we will assume that we are left with the MSSM.

Therefore, a predictive Gauge-Yukawa unified $SU(5)$ model which is finite to all orders, in addition to the requirements mentioned already, should also have the following properties:

1. One-loop anomalous dimensions are diagonal, i.e., $\gamma_i^{(1)\,j} \propto \delta^j_i$.

2. The three fermion generations, in the irreducible representations $\overline{\mathbf{5}}_i, \mathbf{10}_i$ $(i = 1, 2, 3)$, should not couple to the adjoint $\mathbf{24}$.

3. The two Higgs doublets of the MSSM should mostly be made out of a pair of Higgs quintet and anti-quintet, which couple to the third generation.

In the following we discuss two versions of the all-order finite model. The model of Ref. [2], which will be labeled $\mathbf{A}$, and a slight variation of this model (labeled $\mathbf{B}$), which can also be obtained from the class of the models suggested in Ref. [37] with a modification to suppress non-diagonal anomalous dimensions [3].

The superpotential which describes the two models before the reduction of couplings takes places is of the form [2, 30, 31, 36]

$$\begin{aligned} W &= \sum_{i=1}^{3}\Big[\,\frac{1}{2}g_i^u\,\mathbf{10}_i\mathbf{10}_i H_i + g_i^d\,\mathbf{10}_i\overline{\mathbf{5}}_i\,\overline{H}_i\,\Big] + g_{23}^u\,\mathbf{10}_2\mathbf{10}_3 H_4 \\ &\quad + g_{23}^d\,\mathbf{10}_2\overline{\mathbf{5}}_3\,\overline{H}_4 + g_{32}^d\,\mathbf{10}_3\overline{\mathbf{5}}_2\,\overline{H}_4 + \sum_{a=1}^{4} g_a^f\,H_a\,\mathbf{24}\,\overline{H}_a + \frac{g^\lambda}{3}\,(\mathbf{24})^3\ , \end{aligned} \tag{11}$$

	$\overline{\mathbf{5}}_1$	$\overline{\mathbf{5}}_2$	$\overline{\mathbf{5}}_3$	$\mathbf{10}_1$	$\mathbf{10}_2$	$\mathbf{10}_3$	H_1	H_2	H_3	H_4	$\overline{H}_1$	$\overline{H}_2$	$\overline{H}_3$	$\overline{H}_4$	$\mathbf{24}$
Z_7	4	1	2	1	2	4	5	3	6	-5	-3	-6	0	0	0
Z_3	0	0	0	1	2	0	1	2	0	-1	-2	0	0	0	0
Z_2	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0

Table 1: Charges of the $Z_7 \times Z_3 \times Z_2$ symmetry for Model **FUTA**.

where H_a and $\overline{H}_a$ $(a = 1, \ldots, 4)$ stand for the Higgs quintets and anti-quintets.

We will investigate two realizations of the model, labelled **A** and **B**. The main difference between model **A** and model **B** is that two pairs of Higgs quintets and anti-quintets couple to the **24** in **B**, so that it is not necessary to mix them with H_4 and $\overline{H}_4$ in order to achieve the triplet-doublet splitting after the symmetry breaking of $SU(5)$ [3]. Thus, although the particle content is the same, the solutions to Eq.(5) and the sum rules are different, which will reflect in the phenomenology, as we will see.

4.1 FUTA

After the reduction of couplings the symmetry of the superpotential W (11) is enhanced. For model **A** one finds that the superpotential has the $Z_7 \times Z_3 \times Z_2$ discrete symmetry with the charge assignment as shown in Table 1, and with the following superpotential

$$W = \sum_{i=1}^{3} [\, \frac{1}{2} g_i^u \, \mathbf{10}_i \mathbf{10}_i H_i + g_i^d \, \mathbf{10}_i \overline{\mathbf{5}}_i \, \overline{H}_i \,] + g_4^f \, H_4 \, \mathbf{24} \, \overline{H}_4 + \frac{g^\lambda}{3} \, (\mathbf{24})^3 \ , \tag{12}$$

The non-degenerate and isolated solutions to $\gamma_i^{(1)} = 0$ for model **FUTA**, which are the boundary conditions for the Yukawa couplings at the GUT scale, are:

$$\begin{aligned}
&(g_1^u)^2 = \frac{8}{5} \, g^2 \ , \ (g_1^d)^2 = \frac{6}{5} \, g^2 \ , \ (g_2^u)^2 = (g_3^u)^2 = \frac{8}{5} \, g^2 \ , \\
&(g_2^d)^2 = (g_3^d)^2 = \frac{6}{5} \, g^2 \ , \ (g_{23}^u)^2 = 0 \ , \ (g_{23}^d)^2 = (g_{32}^d)^2 = 0 \ , \\
&(g^\lambda)^2 = \frac{15}{7} g^2 \ , \ (g_2^f)^2 = (g_3^f)^2 = 0 \ , \ (g_1^f)^2 = 0 \ , \ (g_4^f)^2 = g^2 \ .
\end{aligned} \tag{13}$$

In the dimensionful sector, the sum rule gives us the following boundary conditions at the GUT scale for this model [30, 31, 36]:

$$m_{H_u}^2 + 2 m_{\mathbf{10}}^2 = m_{H_d}^2 + m_{\overline{\mathbf{5}}}^2 + m_{\mathbf{10}}^2 = M^2 \ , \tag{14}$$

and thus we are left with only three free parameters, namely $m_{\overline{\mathbf{5}}} \equiv m_{\overline{\mathbf{5}}_3}$, $m_{\mathbf{10}} \equiv m_{\mathbf{10}_3}$ and M.

4.2 FUTB

Also in the case of **FUTB** the symmetry is enhanced after the reduction of couplings. The superpotential has now a $Z_4 \times Z_4 \times Z_4$ symmetry with charges as shown in Table 2 and with

	$\overline{\mathbf{5}}_1$	$\overline{\mathbf{5}}_2$	$\overline{\mathbf{5}}_3$	$\mathbf{10}_1$	$\mathbf{10}_2$	$\mathbf{10}_3$	H_1	H_2	H_3	H_4	$\overline{H}_1$	$\overline{H}_2$	$\overline{H}_3$	$\overline{H}_4$	$\mathbf{24}$
Z_4	1	0	0	1	0	0	2	0	0	0	-2	0	0	0	0
Z_4	0	1	0	0	1	0	0	2	0	3	0	-2	0	-3	0
Z_4	0	0	1	0	0	1	0	0	2	3	0	0	-2	-3	0

Table 2: Charges of the $Z_4 \times Z_4 \times Z_4$ symmetry for Model **FUTB**.

the following superpotential

$$\begin{aligned} W &= \sum_{i=1}^{3} [\, \frac{1}{2} g_i^u \, \mathbf{10}_i \mathbf{10}_i H_i + g_i^d \, \mathbf{10}_i \overline{\mathbf{5}}_i \, \overline{H}_i \,] + g_{23}^u \, \mathbf{10}_2 \mathbf{10}_3 H_4 \\ & \quad + g_{23}^d \, \mathbf{10}_2 \overline{\mathbf{5}}_3 \, \overline{H}_4 + g_{32}^d \, \mathbf{10}_3 \overline{\mathbf{5}}_2 \, \overline{H}_4 + g_2^f \, H_2 \, \mathbf{24} \, \overline{H}_2 + g_3^f \, H_3 \, \mathbf{24} \, \overline{H}_3 + \frac{g^\lambda}{3} \, (\mathbf{24})^3 \ , \end{aligned} \tag{15}$$

For this model the non-degenerate and isolated solutions to $\gamma_i^{(1)} = 0$ give us:

$$\begin{aligned} & (g_1^u)^2 = \frac{8}{5} \, g^2 \ , \ (g_1^d)^2 = \frac{6}{5} \, g^2 \ , \ (g_2^u)^2 = (g_3^u)^2 = \frac{4}{5} \, g^2 \ , \\ & (g_2^d)^2 = (g_3^d)^2 = \frac{3}{5} \, g^2 \ , \ (g_{23}^u)^2 = \frac{4}{5} \, g^2 \ , \ (g_{23}^d)^2 = (g_{32}^d)^2 = \frac{3}{5} \, g^2 \ , \\ & (g^\lambda)^2 = \frac{15}{7} g^2 \ , \ (g_2^f)^2 = (g_3^f)^2 = \frac{1}{2} \, g^2 \ , \ (g_1^f)^2 = 0 \ , \ (g_4^f)^2 = 0 \ , \end{aligned} \tag{16}$$

and from the sum rule we obtain:

$$\begin{aligned} m_{H_u}^2 + 2 m_{\mathbf{10}}^2 &= M^2 \ , \ m_{H_d}^2 - 2 m_{\mathbf{10}}^2 = -\frac{M^2}{3} \ , \\ m_{\overline{\mathbf{5}}}^2 + 3 m_{\mathbf{10}}^2 &= \frac{4M^2}{3} \ , \end{aligned} \tag{17}$$

i.e., in this case we have only two free parameters $m_{\mathbf{10}} \equiv m_{\mathbf{10}_3}$ and M for the dimensionful sector.

As already mentioned, after the $SU(5)$ gauge symmetry breaking we assume we have the MSSM, i.e. only two Higgs doublets. This can be achieved by introducing appropriate mass terms that allow to perform a rotation of the Higgs sector [2, 30–32], in such a way that only one pair of Higgs doublets, coupled mostly to the third family, remains light and acquire vacuum expectation values. To avoid fast proton decay the usual fine tuning to achieve doublet-triplet splitting is performed. Notice that, although similar, the mechanism is not identical to minimal $SU(5)$, since we have an extended Higgs sector.

Thus, after the gauge symmetry of the GUT theory is broken we are left with the MSSM, with the boundary conditions for the third family given by the finiteness conditions, while the other two families are basically decoupled.

We will now examine the phenomenology of such all-loop Finite Unified theories with $SU(5)$ gauge group and, for the reasons expressed above, we will concentrate only on the third generation of quarks and leptons. An extension to three families, and the generation of

quark mixing angles and masses in Finite Unified Theories has been addressed in [38], where several examples are given. These extensions are not considered here. Realistic Finite Unified Theories based on product gauge groups, where the finiteness implies three generations of matter, have also been studied [39].

5 Restrictions from the low-energy observables

Since the gauge symmetry is spontaneously broken below $M_{\rm GUT}$, the finiteness conditions do not restrict the renormalization properties at low energies, and all it remains are boundary conditions on the gauge and Yukawa couplings (13) or (16), the $h = -MC$ relation (7), and the soft scalar-mass sum rule (8) at $M_{\rm GUT}$, as applied in the two models. Thus we examine the evolution of these parameters according to their RGEs up to two-loops for dimensionless parameters and at one-loop for dimensionful ones with the relevant boundary conditions. Below $M_{\rm GUT}$ their evolution is assumed to be governed by the MSSM. We further assume a unique supersymmetry breaking scale $M_{\rm SUSY}$ (which we define as the geometrical average of the stop masses) and therefore below that scale the effective theory is just the SM. This allows to evaluate observables at or below the electroweak scale.

In the following, we briefly describe the low-energy observables used in our analysis. We discuss the current precision of the experimental results and the theoretical predictions. We also give relevant details of the higher-order perturbative corrections that we include. We do not discuss theoretical uncertainties from the RG running between the high-scale parameters and the weak scale. At present, these uncertainties are expected to be less important than the experimental and theoretical uncertainties of the precision observables.

As precision observables we first discuss the 3rd generation quark masses that are leading to the strongest constraints on the models under investigation. Next we apply B physics and Higgs-boson mass constraints. Parameter points surviving these constraints are then tested with the cold dark matter (CDM) abundance in the early universe. We also briefly discuss the anomalous magnetic moment of the muon.

5.1 The quark masses

Since the masses of the (third generation) quarks are no free parameters in our model but predicted in terms of the GUT scale parameters and the τ mass, m_t and m_b are (as it turns out the most restrictive) precision observables for our analysis. For the top-quark mass we use the current experimental value for the pole mass [40]

$$m_t^{\rm exp} = 170.9 \pm 1.8 \text{ GeV} . \tag{18}$$

For the bottom-quark mass we use the value at the bottom-quark mass scale or at M_Z [41]

$$\overline{m}_b(m_b) = 4.25 \pm 0.1 \text{ GeV} \quad \text{or} \quad \overline{m}_b(M_Z) = 2.82 \pm 0.07 \text{ GeV} . \tag{19}$$

It should be noted that a numerically important correction appears in the relation between the bottom-quark mass and the bottom Yukawa coupling (that also enters the corresponding RGE running). The leading $\tan\beta$-enhanced corrections arise from one-loop contributions

with gluino-sbottom and chargino-stop loops. We include the leading effects via the quantity Δ_b [42] (see also Refs. [43–45]). Numerically the correction to the relation between the bottom-quark mass and the bottom Yukawa coupling is usually by far the dominant part of the contributions from the sbottom sector (see also Refs. [46, 47]). In the limit of large soft SUSY-breaking parameters and $\tan\beta \gg 1$, Δ_b is given by [42]

$$\Delta_b = \frac{2\alpha_s}{3\,\pi}\, m_{\tilde{g}}\, \mu\, \tan\beta \, \times \, I(m_{\tilde{b}_1}, m_{\tilde{b}_2}, m_{\tilde{g}}) + \frac{\alpha_t}{4\,\pi}\, A_t\, \mu\, \tan\beta \, \times \, I(m_{\tilde{t}_1}, m_{\tilde{t}_2}, |\mu|) \;, \qquad (20)$$

where the gluino mass is denoted by $m_{\tilde{g}}$ and $\alpha_f \equiv h_f^2/(4\,\pi)$, h_f being a fermion Yukawa coupling. The function I is defined as

$$\begin{aligned} I(a,b,c) &= \frac{1}{(a^2-b^2)(b^2-c^2)(a^2-c^2)} \left(a^2 b^2 \log\frac{a^2}{b^2} + b^2 c^2 \log\frac{b^2}{c^2} + c^2 a^2 \log\frac{c^2}{a^2} \right) \qquad (21) \\ &\sim \frac{1}{\max(a^2, b^2, c^2)} \;. \end{aligned}$$

A corresponding correction of $\mathcal{O}(\alpha_\tau)$ has been included for the relation between the τ lepton mass and the τ Yukawa coupling. However, this correction is much smaller than the one given in eq. (20).

The Δ_b corrections are included by the replacement

$$\overline{m}_b \;\to\; \frac{\overline{m}_b}{1+\Delta_b} \;, \qquad (22)$$

resulting in a resummation of the leading terms in $\mathcal{O}(\alpha_s \tan\beta)$ and $\mathcal{O}(\alpha_t \tan\beta)$ to all-orders. Expanding eq. (22) to first or second order gives an estimate of the effect of the resummation of the Δ_b terms and has been used as an estimate of unknown higher-order corrections (see below).

5.2 The decay $b \to s\gamma$

Since this decay occurs at the loop level in the SM, the MSSM contribution might *a priori* be of similar magnitude. A recent theoretical estimate of the SM contribution to the branching ratio at the NNLO QCD level is [48]

$$\mathrm{BR}(b \to s\gamma) = (3.15 \pm 0.23) \times 10^{-4} \;. \qquad (23)$$

It should be noted that the error estimate for $\mathrm{BR}(b \to s\gamma)$ is still under discussion [49], and that other SM contributions to $b \to s\gamma$ have been calculated [50]. These corrections are small compared with the theoretical uncertainty quoted in eq. (23).

For comparison, the present experimental value estimated by the Heavy Flavour Averaging Group (HFAG) is [51, 52]

$$\mathrm{BR}(b \to s\gamma) = (3.55 \pm 0.24^{+0.09}_{-0.10} \pm 0.03) \times 10^{-4}, \qquad (24)$$

where the first error is the combined statistical and uncorrelated systematic uncertainty, the latter two errors are correlated systematic theoretical uncertainties and corrections respectively.

Our numerical results have been derived with the BR($b \to s\gamma$) evaluation provided in Refs. [53–55], incorporating also the latest SM corrections provided in Ref. [48]. The calculation has been checked against other codes [56–58]. Concerning the total error in a conservative approach we add linearly the errors of eqs. (23) and (24) as well an intrinsic SUSY error of 0.15×10^{-4} [25].

5.3 The decay $B_s \to \mu^+\mu^-$

The SM prediction for this branching ratio is $(3.4 \pm 0.5) \times 10^{-9}$ [59], and the present experimental upper limit from the Fermilab Tevatron collider is 5.8×10^{-8} at the 95% C.L. [60], still providing the possibility for the MSSM to dominate the SM contribution. The current Tevatron sensitivity, being based on an integrated luminosity of about 2 fb^{-1}, is expected to improve somewhat in the future. In Ref. [60] an estimate of the future Tevatron sensitivity of 2×10^{-8} at the 90% C.L. has been given, and a sensitivity even down to the SM value can be expected at the LHC. Assuming the SM value, i.e. BR($B_s \to \mu^+\mu^-$) $\approx 3.4 \times 10^{-9}$, it has been estimated [61] that LHCb can observe 33 signal events over 10 background events within 3 years of low-luminosity running. Therefore this process offers good prospects for probing the MSSM.

For the theoretical prediction we use the code implemented in Ref. [56] (see also Ref. [62]), which includes the full one-loop evaluation and the leading two-loop QCD corrections. We are not aware of a detailed estimate of the theoretical uncertainties from unknown higher-order corrections.

5.4 The lightest MSSM Higgs boson mass

The mass of the lightest $\mathcal{CP}$-even MSSM Higgs boson can be predicted in terms of the other SUSY parameters. At the tree level, the two $\mathcal{CP}$-even Higgs boson masses are obtained as a function of M_Z, the $\mathcal{CP}$-odd Higgs boson mass M_A, and $\tan\beta$. We employ the Feynman-diagrammatic method for the theoretical prediction of M_h, using the code `FeynHiggs` [63–66], which includes all relevant higher-order corrections. The status of the incorporated results can be summarized as follows. For the one-loop part, the complete result within the MSSM is known [67, 68]. Concerning the two-loop effects, their computation is quite advanced, see Ref. [65] and references therein. They include the strong corrections at $\mathcal{O}(\alpha_t\alpha_s)$ and Yukawa corrections at $\mathcal{O}(\alpha_t^2)$ to the dominant one-loop $\mathcal{O}(\alpha_t)$ term, and the strong corrections from the bottom/sbottom sector at $\mathcal{O}(\alpha_b\alpha_s)$. For the $b/\tilde{b}$ sector corrections also an all-order resummation of the $\tan\beta$-enhanced terms, $\mathcal{O}(\alpha_b(\alpha_s \tan\beta)^n)$, is known. The current intrinsic error of M_h due to unknown higher-order corrections have been estimated to be [65, 69–71]

$$\Delta M_h^{\rm intr,current} = 3 \text{ GeV}. \tag{25}$$

The lightest MSSM Higgs boson is the models under consideration is always SM-like (see also Refs. [72, 73]). Consequently, the current LEP bound of $M_h^{\rm exp} > 114.4$ GeV at the 95% C.L. can be taken over [20, 21].

5.5 Cold dark matter density

Finally we discuss the impact of the cold dark matter (CDM) density. It is well known that the lightest neutralino, being the lightest supersymmetric particle (LSP), is an excellent candidate for CDM [23]. Consequently we demand that the lightest neutralino is indeed the LSP. Parameters leading to a different LSP are discarded.

The current bound, favored by a joint analysis of WMAP and other astrophysical and cosmological data [22], is at the $2\,\sigma$ level given by the range

$$0.094 < \Omega_{\rm CDM} h^2 < 0.129 \; . \tag{26}$$

Assuming that the cold dark matter is composed predominantly of LSPs, the determination of $\Omega_{\rm CDM} h^2$ imposes very strong constraints on the MSSM parameter space. As will become clear below, no model points fulfill the strict bound of eq. (26). On the other hand, many model parameters would yield a very large value of $\Omega_{\rm CDM}$. It should be kept in mind that somewhat larger values might be allowed due to possible uncertainties in the determination of the SUSY spectrum (as they might arise at large $\tan\beta$, see below).

However, on a more general basis and not speculating about unknown higher-order uncertainties, a mechanism is needed in our model to reduce the CDM abundance in the early universe. This issue could, for instance, be related to another problem, that of neutrino masses. This type of masses cannot be generated naturally within the class of finite unified theories that we are considering in this paper, although a non-zero value for neutrino masses has clearly been established [41]. However, the class of FUTs discussed here can, in principle, be easily extended by introducing bilinear R-parity violating terms that preserve finiteness and introduce the desired neutrino masses [102]. R-parity violation [103] would have a small impact on the collider phenomenology presented here (apart from fact the SUSY search strategies could not rely on a 'missing energy' signature), but remove the CDM bound of eq. (26) completely. The details of such a possibility in the present framework attempting to provide the models with realistic neutrino masses will be discussed elsewhere. Other mechanisms, not involving R-parity violation (and keeping the 'missing energy' signature), that could be invoked if the amount of CDM appears to be too large, concern the cosmology of the early universe. For instance, "thermal inflation" [74] or "late time entropy injection" [75] could bring the CDM density into agreement with the WMAP measurements. This kind of modifications of the physics scenario neither concerns the theory basis nor the collider phenomenology, but could have a strong impact on the CDM derived bounds.

Therefore, in order to get an impression of the *possible* impact of the CDM abundance on the collider phenomenology in our models under investigation, we will analyze the case that the LSP does contribute to the CDM density, and apply a more loose bound of

$$\Omega_{\rm CDM} h^2 < 0.3 \; . \tag{27}$$

(Lower values than the ones permitted by eq. (26) are naturally allowed if another particle than the lightest neutralino constitutes CDM.) For our evaluation we have used the code `MicroMegas` [56].

5.6 The anomalous magnetic moment of the muon

We finally comment on the status and the impact of the anomalous magnetic moment of the muon, $a_\mu \equiv \frac{1}{2}(g-2)_\mu$. The SM prediction for a_μ (see Refs. [76–79] for reviews) depends on the evaluation of QED contributions, the hadronic vacuum polarization and light-by-light (LBL) contributions. The evaluations of the hadronic vacuum polarization contributions using e^+e^- and τ decay data give somewhat different results. The latest estimate based on e^+e^- data [80] is given by:

$$a_\mu^{\rm theo} = (11\,659\,180.5 \pm 4.4_{\rm had} \pm 3.5_{\rm LBL} \pm 0.2_{\rm QED+EW}) \times 10^{-10}, \tag{28}$$

where the source of each error is labeled. We note that the new e^+e^- data sets that have recently been published in Refs. [81–83] have been partially included in the updated estimate of $(g-2)_\mu$.

The SM prediction is to be compared with the final result of the Brookhaven $(g-2)_\mu$ experiment E821 [84], namely:

$$a_\mu^{\rm exp} = (11\,659\,208.0 \pm 6.3) \times 10^{-10}, \tag{29}$$

leading to an estimated discrepancy [80, 85]

$$a_\mu^{\rm exp} - a_\mu^{\rm theo} = (27.5 \pm 8.4) \times 10^{-10}, \tag{30}$$

equivalent to a 3.3-σ effect (see also Refs. [78, 86, 87]). In order to illustrate the possible size of corrections, a simplified formula can be used, in which relevant supersymmetric mass scales are set to a common value, $M_{\rm SUSY} = m_{\tilde\chi^\pm} = m_{\tilde\chi^0} = m_{\tilde\mu} = m_{\tilde\nu_\mu}$. The result in this approximation is given by

$$a_\mu^{\rm SUSY,1L} = 13 \times 10^{-10} \left(\frac{100\ \rm GeV}{M_{\rm SUSY}}\right)^2 \tan\beta\ {\rm sign}(\mu). \tag{31}$$

It becomes obvious that $\mu < 0$ is already challenged by the present data on a_μ. However, a heavy SUSY spectrum with $\mu < 0$ results in a $a_\mu^{\rm SUSY}$ prediction very close to the SM result. Since the SM is not regarded as excluded by $(g-2)_\mu$, we also still allow both signs of μ in our analysis.

Concerning the MSSM contribution, the complete one-loop result was evaluated a decade ago [88]. In addition to the full one-loop contributions, the leading QED two-loop corrections have also been evaluated [89]. Further corrections at the two-loop level have been obtained in Refs. [90, 91], leading to corrections to the one-loop result that are $\lesssim 10\%$. These corrections are taken into account in our analysis according to the approximate formulae given in Refs. [90, 91].

6 Final Predictions

In this section we present the predictions of the models **FUTA** and **FUTB** with ($\mu > 0$ and $\mu < 0$), whose theoretically restricted parameter space due to finiteness has been further reduced by requiring correct electroweak symmetry breaking and the absence of charge or color

breaking minima. We furthermore demand that the bounds discussed in the previous section are also fulfilled, see the following subsections. We have performed a scan over the GUT scale parameters, where we take as further input the τ mass, $m_\tau = 1.777$ GeV. This allows us to extract the value of v_u, and then, using the relation $M_Z^2 = \frac{1}{2}\sqrt{(3g_1^2/5 + g_2^2)(v_u^2 + v_d^2)}$, $v_{u,d} = 1/\sqrt{2}\langle H_{u,d}\rangle$, we can extract the value of v_d. In this way it is possible to predict the masses of the top and bottom quarks, and the value of $\tan\beta$. As already mentioned, we take into account the large radiative corrections to the bottom mass, see eq. (22), as well as the ones to the tau mass. We have furthermore estimated the corrections to the top mass in our case and found them to be negligible, so they are not included in our analysis. As a general result for both models and both signs of μ we have a heavy SUSY mass spectrum, and $\tan\beta$ always has a large value of $\tan\beta \sim 44 - 56$.

6.1 Results vs. quark masses

The first low-energy constraint applied are the top- and bottom-quark masses as given in Sect. 5.1. In Fig. 1 we present the predictions of the models concerning the bottom quark mass. The steps in the values for **FUTA** are due to the fact that fixed values of M were taken, while the other parameters m_5 and m_{10} were varied. However, this selected sampling of the parameter space is sufficient for us to draw our conclusions, see below.

We present the predictions for $\overline{m}_b(M_Z)$, to avoid unnecessary errors coming from the running from M_Z to the m_b pole mass, which are not related to the predictions of the present models. As already mentioned in section 5.1, we estimated the effect of the unknown higher order corrections. For such large values of $\tan\beta$, see above, in the case of **FUTB** for the bottom mass they are $\sim 8\%$, whereas for **FUTA** they can go to $\sim 30\%$ (these uncertainties are slightly larger for $\mu > 0$ than for $\mu < 0$). Although these theoretical uncertainties are not shown in the graphs, they have been taken into the account in the analysis of $\overline{m}_b$, by selecting only the values that comply with the value of the bottom mass within this theoretical error.

From the bounds on the $\overline{m}_b(M_Z)$ mass, we can see from Fig. 1 that the region $\mu > 0$ is excluded both for **FUTA** and **FUTB** while for $\mu < 0$ both models lie partially within the experimental limits.

In Fig. 2 we present the predictions of the models **FUTA** and **FUTB** concerning the top quark pole mass. We recall that the theoretical predictions of m_t have an uncertainty of $\sim 4\%$ [92]. The current experimental value is given in eq. (18). This clearly favors **FUTB** while **FUTA** corresponds to m_t values that are somewhat outside the experimental range, even taking theoretical uncertainties into account. Thus m_t and $\overline{m}_b(M_Z)$ together single out **FUTB** with $\mu < 0$ as the most favorable solution. From Sect. 5.6 it is obvious that $\mu < 0$ is already challenged by the present data on a_μ. However, a heavy SUSY spectrum as we have here (see above and Sect. 6.3) with $\mu < 0$ results in a $a_\mu^{\rm SUSY}$ prediction very close to the SM result. Since the SM is not regarded as excluded by $(g-2)_\mu$, we continue with our analysis of **FUTB** with $\mu < 0$.

6.2 Results for precision observables and CDM

For the remaining model, **FUTB** with $\mu < 0$, we compare the predictions for BR($b \to s\gamma$), BR($B_s \to \mu^+\mu^-$) and M_h with their respective experimental constraints, see Sects. 5.2 –

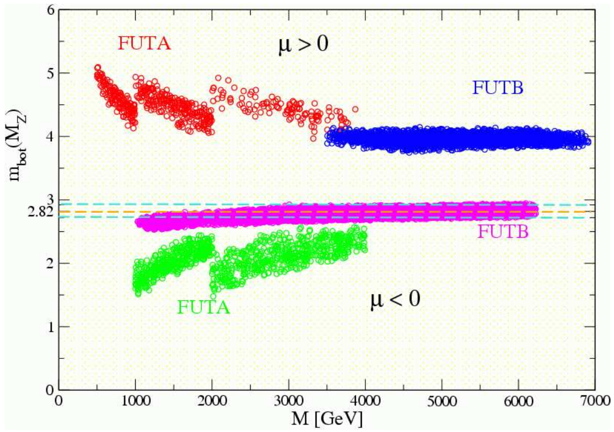

Figure 1: $\overline{m}_b(M_Z)$ as function of M for models **FUTA** and **FUTB**, for $\mu < 0$ and $\mu > 0$.

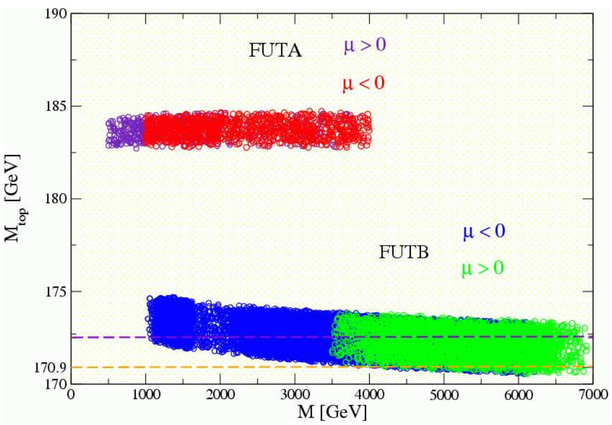

Figure 2: m_t as function of M for models **FUTA** and **FUTB**, for $\mu < 0$ and $\mu > 0$.

5.4. First, in Fig. 3 we show the predictions for BR($b \to s\gamma$) vs. BR($B_s \to \mu^+\mu^-$) for all the points of **FUTB** with $\mu < 0$. The gray (red) points in the lower left corner fulfill the B physics constraints as given in Sects. 5.2, 5.3. Shown also in black are the parameter points that fulfill the loose CDM constraint of eq. (27), which can be found in the whole B physics allowed area.

In the second step we test the compatibility with the Higgs boson mass constraints and the CDM bounds. In Fig. 4 we show M_h (as evaluated with `FeynHiggs` [63–66]) as a function of M for **FUTB** with $\mu < 0$. Only the points that also fulfill the B physics bounds are included. The prediction for the Higgs boson mass is constrained to the interval $M_h = 118 \ldots 129$ GeV (including the intrinsic uncertainties of eq. (25)), thus fulfilling automatically the LEP bounds [20, 21]. Furthermore indicated in Fig. 4 by the darker (red) points is the parameter space that in addition fulfills the CDM constraint as given in eq. (27). The loose bound permits values of M from ~ 1000 GeV to about ~ 3000 GeV. The strong CDM

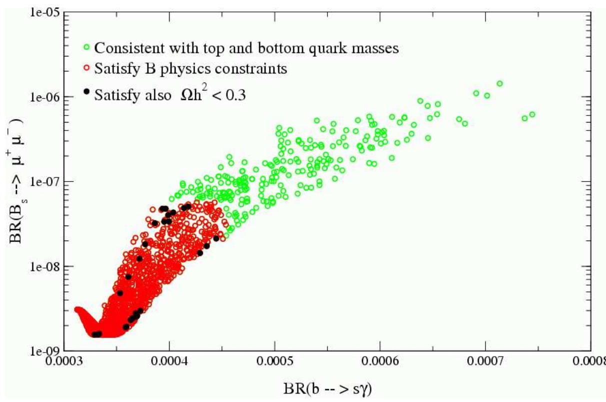

Figure 3: BR($b \to s\gamma$) vs. BR($B_s \to \mu^+\mu^-$). In green (light gray) are the points consistent with the top and bottom quark masses, in red (gray) are the subset of these that fulfill the B physics constraints, and in black the ones that also satisfy the CDM loose constraint.

bound, eq. (26), on the other hand, is not fulfilled by any data point, where the points with lowest $\Omega_{\rm CDM}h^2 \sim 0.2$ can be found for $M \gtrsim 1500$ GeV. As mentioned in Sect. 5.5, the CDM bounds should be viewed as "additional" constraints (when investigating the collider phenomenology). But even taking eq. (27) at face value, due to possible larger uncertainties in the calculation of the SUSY spectrum as outlined above, the CDM constraint (while strongly reducing the allowed parameter space) does not exclude the model. Within the current calculation data points which are in strict agreement with eq. (26) violate the B physics constraints.

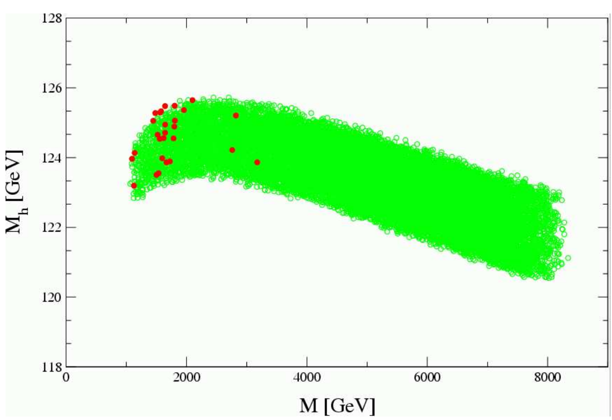

Figure 4: M_h is shown as a function of M. The light (green) points fulfill the B physics constraints. The darker (red) dots in addition satisfy the loose CDM constraint of eq. (27).

6.3 The heavy Higgs and SUSY spectrum

The gray (red) points shown in Fig. 3 are the prediction of the finite theories once confronted with low-energy experimental data. In order to assess the discovery potential of the LHC [93, 94] and/or the ILC [95–98] we show the corresponding predictions for the most relevant SUSY mass parameters. In Fig. 5 we plot the mass of the lightest observable SUSY particle (LOSP) as function of M, that comply with the B physics constraints, as explained above. The darker (red) points fulfill in addition the loose CDM constraint eq. (27). The LOSP is either the light scalar τ or the second lightest neutralino (which is close in mass with the lightest chargino). One can see that the masses are outside the reach of the LHC and also the ILC. Neglecting the CDM constraint, even higher particle masses are allowed.

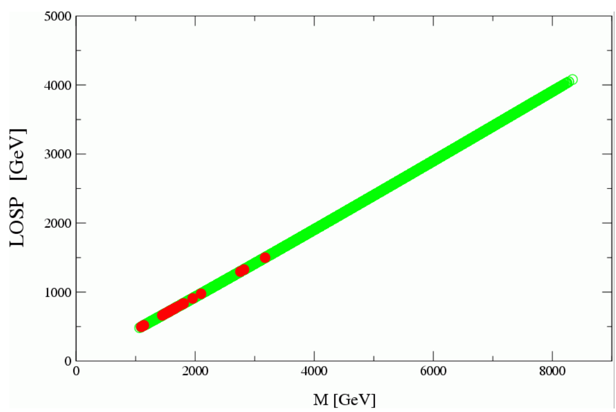

Figure 5: The mass of the LOSP is presented as a function of M. Shown are only points that fulfill the B physics constraints. The dark (red) dots in addition also satisfy the loose CDM constraint of eq. (27).

More relevant for the LHC are the colored particles. Therefore, in Fig. 6 we show the masses of various colored particles: $m_{\tilde{t}_1}$, $m_{\tilde{b}_1}$ and $m_{\tilde{g}}$. The masses show a nearly linear dependence on M. Assuming a discovery reach of ~ 2.5 TeV yields a coverage up to $M \lesssim 2$ TeV. This corresponds to the largest part of the CDM favored parameter space. All these particles are outside the reach of the ILC. Disregarding the CDM bounds, see Sect. 5.5, on the other hand, results in large parts of the parameter space in which no SUSY particle can be observed neither at the LHC nor at the ILC.

We now turn to the predictions for the Higgs boson sector of **FUTB** with $\mu < 0$. In Fig. 7 we present the prediction for M_h vs. M_A, with the same color code as in Fig. 5. We have truncated the plot at about $M_A = 10$ TeV. The parameter space allowed by B physics extends up to ~ 30 TeV. The values that comply with the CDM constraints are in a relatively light region of M_A with $M_A \lesssim 4000$ GeV. However, taking Figs. 4 and 7 into account, the LHC and the ILC will observe only a light Higgs boson, whereas the heavy Higgs bosons remain outside the LHC or ILC reach.

There might be the possibility to distinguish the light MSSM Higgs boson from the SM

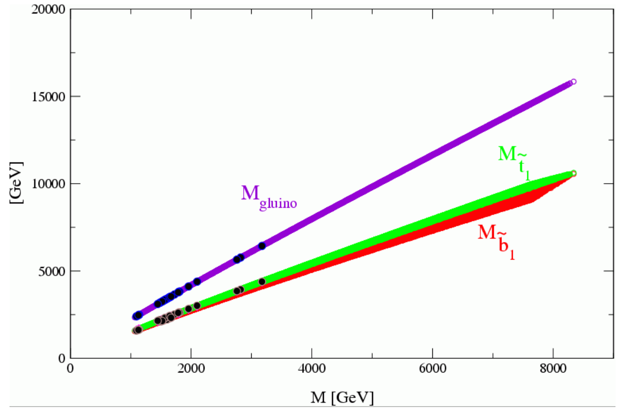

Figure 6: The mass of various colored particles are presented as a function of M. Shown are only points that fulfill the B physics constraints, the black ones satisfy also the loose CDM constraint.

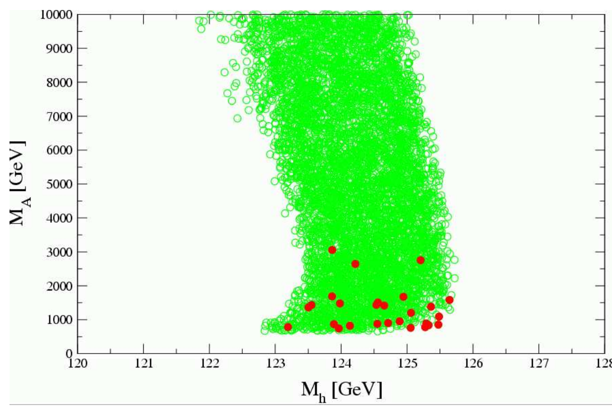

Figure 7: M_A vs M_h, with the same color code as in Fig. 5.

Higgs boson by its decay characteristics. It has been shown that the ratio

$$\frac{\mathrm{BR}(h \to b\bar{b})}{\mathrm{BR}(h \to WW^*)} \tag{32}$$

is the most powerful discriminator between the SM and the MSSM using ILC measurements [99,100]. We assume an experimental resolution of this ratio of $\sim 1.5\%$ at the ILC [101]. In Fig. 8 we show the ratio as a function of M with the same color code as in Fig. 5. It can be seen that up to $M \lesssim 2$ TeV a deviation from the SM ratio of more than $3\,\sigma$ can be observed. This covers most of the CDM favored parameter space. Neglecting the CDM constraint, i.e. going to higher values of M, results in a light Higgs boson that is indistinguishable from a SM Higgs boson.

Finally, in Tab. 3 we present a representative example of the values obtained for the SUSY and Higgs boson masses for Model **FUTB** with $\mu < 0$. The masses are typically large, as

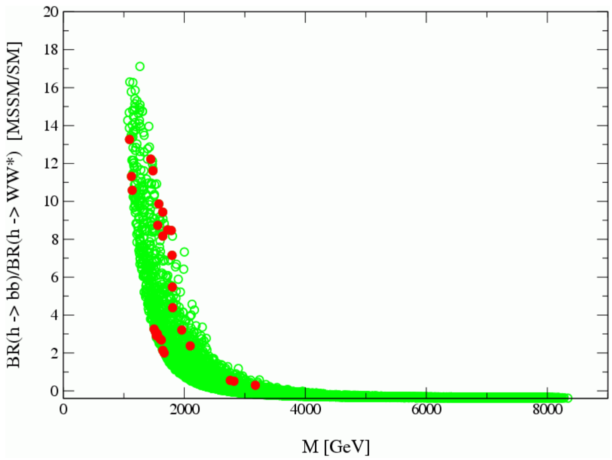

Figure 8: $\mathrm{BR}(h \to b\bar{b})/\mathrm{BR}(h \to WW^*)$ [MSSM/SM] (expressed in terms of σ with a resolution of 1.5% (see text)) is shown as a function of M. The color code is the same as in Fig. 5.

already mentioned, with the LOSP starting from $\gtrsim 1000$ GeV.

It should be kept in mind that although we present the results that are consistent with the (loose) CDM constraints, the present model considers only the third generation of (s)quarks and (s)leptons. A more complete analysis will be given elsewhere when flavor mixing will be taken into account, see e.g. Ref. [38]. A similar remark concerns the neutrino masses and mixings. It is well known that they can be introduced via bilinear R-parity violating terms [103] which preserve finiteness. In this case the dark matter candidate will not be the lightest neutralino, but could be another one, e.g. the axion.

7 Conclusions

In the present paper we have examined the predictions of two $N = 1$ supersymmetric and moreover all-loop finite $SU(5)$ unified models, leading after the spontaneous symmetry breaking at the Grand Unification scale to the *finiteness-constrained MSSM*.

The finiteness conditions in the supersymmetric part of the unbroken theory lead to relations among the dimensionless couplings, i.e. *gauge-Yukawa unification*. In addition the finiteness conditions in the SUSY-breaking sector of the theories lead to a tremendous reduction of the number of the independent soft SUSY-breaking parameters leaving one model (**A**) with three and another (**B**) with two free parameters. Therefore the *finiteness-constrained MSSM* consists of the well known MSSM with boundary conditions at the Grand Unification scale for its various dimensionless and dimensionful parameters inherited from the all-loop finiteness unbroken theories. Obviously these lead to an extremely restricted and, consequently, very predictive parameter space of the MSSM.

In the present paper the finiteness constrained parameter space of MSSM is confronted with the existing low-energy phenomenology such as the top and bottom quark masses,

m_t	172	$\overline{m}_b(M_Z)$	2.7
$\tan\beta =$	46	α_s	0.116
$m_{\tilde{\chi}^0_1}$	796	$m_{\tilde{\tau}_2}$	1268
$m_{\tilde{\chi}^0_2}$	1462	$m_{\tilde{\nu}_3}$	1575
$m_{\tilde{\chi}^0_3}$	2048	μ	-2046
$m_{\tilde{\chi}^0_4}$	2052	B	4722
$m_{\tilde{\chi}^\pm_1}$	1462	M_A	870
$m_{\tilde{\chi}^\pm_2}$	2052	$M_{H^\pm}$	875
$m_{\tilde{t}_1}$	2478	M_H	869
$m_{\tilde{t}_2}$	2804	M_h	124
$m_{\tilde{b}_1}$	2513	M_1	796
$m_{\tilde{b}_2}$	2783	M_2	1467
$m_{\tilde{\tau}_1}$	798	M_3	3655

Table 3: A representative spectrum of **FUTB** with $\mu < 0$. All masses are in GeV.

B physics observables, the bound on the lightest Higgs boson mass and constraints from the cold dark matter abundance in the universe. In the first step the result of our parameter scan of the finiteness restricted parameter space of MSSM, after applying the quark mass constraints and including theoretical uncertainties at the unification scale, singles out the *finiteness-constrained MSSM* coming from the model (**B**) with $\mu < 0$ (yielding $(g-2)_\mu$ values similar to the SM). This model was further restricted by applying the B physics constraints. The remaining parameter space then automatically fulfills the LEP bounds on the lightest MSSM Higgs boson with $M_h = 118 \ldots 129$ GeV (including already the intrinsic uncertainties). In the final step the CDM measurements have been imposed. Considering the CDM constraints it should be kept in mind that modifications in the model are possible (non-standard cosmology or R-parity violating terms that preserve finiteness) that would have only a small impact on the collider phenomenology. Therefore the CDM relic abundance should be considered as an "additional" constraint, indicating its *possible* impact. In general, a relatively heavy SUSY and Higgs spectrum at the few TeV level has been obtained, where the lower range of masses yield better agreement with the CDM constraint. The mass of the lightest observable SUSY particle (the lightest slepton or the second lightest neutralino) is larger than 500 GeV, which remains unobservable at the LHC and the ILC. The charged SUSY particles start at around 1.5 TeV and grow nearly linearly with M. Large parts of the CDM favored region results in masses of stops and sbottoms below ~ 2.5 TeV and thus might be detectable at the LHC. The measurement of branching ratios of the lightest Higgs boson to bottom quarks and W bosons at the ILC shows a deviation to the SM results of more than $3\,\sigma$ for values of $M \lesssim 2.5$ TeV, again covering most of the CDM favored region.

In conclusion, **FUTB** with $\mu < 0$, fulfilling the existing constraints from quark masses, B physics observables, Higgs boson searches and CDM measurements, results at a heavy

SUSY spectrum and large $\tan\beta$. Nonetheless, colored particles are likely to be observed in the range of ~ 2 TeV at the LHC. The ILC could measure a deviation in the branching ratios of the lightest Higgs boson. However, neglecting the CDM constraint allows larger values of M. This results in a heavier SUSY spectrum, outside the reach of the LHC and the ILC. In this case also the lightest Higgs boson is SM-like.

Acknowledgements

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